Engineering Notes

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Inertial Coordinate Representation of Mission Analysis Problems

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Introduction

THIS note describes the use of celestial coordinates for attitude sensor problems. Usually celestial coordinates are used for the orbit representation, and sensor problems are worked in a local sphere centered at the spacecraft. The coupling of the orbit and the sensor geometry is obtained through the classical coordinate transformations. In the present work, it is shown that it is possible to have a unique representation which gives a global picture of the problem. This technique is applied to the attitude determination of a communication satellite and the mission planning of a space telescope.

Graphic Representation in Inertial Coordinates

First, consider the classical celestial sphere defined by its north pole N and the vernal point γ. On this celestial sphere, the usual celestial objects—Sun, Moon, and stars—are defined by their celestial coordinates: the right ascension α and the declination δ. The orbit of a spacecraft is defined as a great circle and its position S as (α_s, δ_s). This is the usual representation encountered in textbooks and papers. When sensor or attitude problems are to be solved, this representation is replaced by another in which the spacecraft is at the center of a local sphere with the relative directions of spin axis, Sun, and the path of scan plotted.[1] However, in this new representation the orbit is generally omitted, or the inertial frame of reference is absent.

An improvement of the past representations can be achieved in the following way. Instead of defining a local sphere on which only the relative directions are plotted, the classical celestial sphere is used, and the relative directions are plotted as if the spacecraft were located at the center of the celestial sphere. Thus all the information related to the locations of celestial objects, the orbit and the spacecraft, has been obtained along with the relative directions. The nadir direction is readily obtained: OS gives the geocentric direction of the spacecraft; $O\bar{S}$ (where $\bar{S}$ is opposite to S) is the nadir direction, and $\bar{S}$ location is simply $\alpha_s + \pi$, $-\delta_s$. The apparent Earth as seen from the spacecraft is a disk with radius η_0 centered on point $\bar{S}$ ($\eta_0 = \sin^{-1} R_e/R$ where R_e is the Earth radius and R the radial distance of the spacecraft). Figure 1 shows the complete geometry and contains all the information necessary for any

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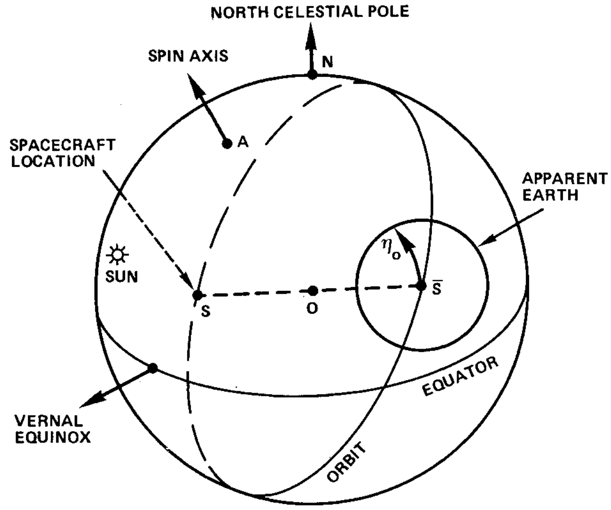

Fig. 1 Celestial coordinate sphere with relative directions.

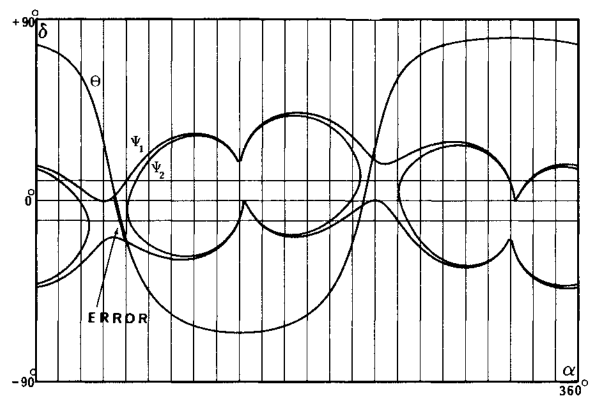

Fig. 2 Plane inertial coordinate representation.

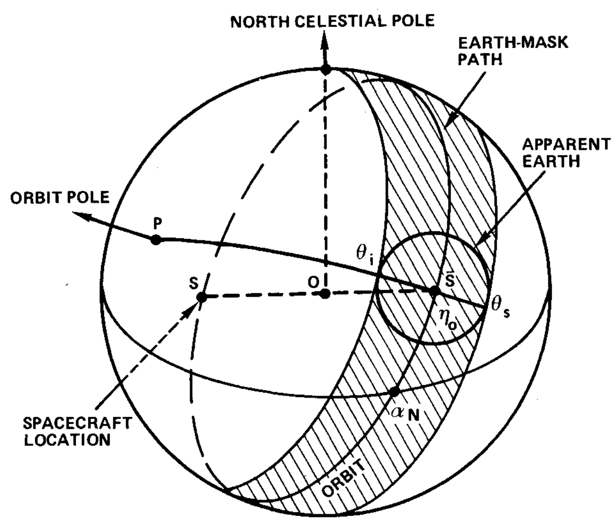

Fig. 3 Celestial sphere with Earth-mask path.

mission analysis, including Earth's mask for a space telescope, scan of the apparent Earth by an infrared sensor, and a scan of the stars combined with other mission constraints. Following are two applications of this representation.

Geometrical Limitations on Attitude Determination of a Spinning Spacecraft

The study is restricted to the case of a spacecraft located at synchronous altitude with a spin axis in the equatorial region. The infrared sensor gives the Sun-spin axis-to-Earth rotation angle Ψ, the Earth half-width angle Λ, and the Sun angle β. This information can be used to determine the location of the spin axis direction in at least two ways.[1] To illustrate the inertial coordinate representation, the analysis will be restricted as follows: the spin axis A is at distance β from the Sun and thus on a circle Θ of center H and radius β. The on-board sensors are of limited accuracy, and the angles β, Ψ are known with some uncertainty. Curves Θ and Ψ should be replaced by strips, and the spin axis is located at the intersection of two strips somewhere inside a distorted parallelogram. A plane representation of the celestial sphere is chosen with α along the x axis and declination δ along the y axis. As the quantities β and Ψ are calculated in the (α, δ) coordinate system, curves Θ of constant β values and curves of constant Ψ are plotted. Figure 2 represents the geometry of the intersection. The strips in Θ and Ψ are obtained by tracing curves corresponding to the upper and lower limits for each variable. The Sun elevation angle β is known with a good accuracy, and the width of the strip is almost negligible. On the other hand, Ψ is known with some error, and the strip is quite wide in the equatorial region. The parallelogram reduces to a segment AB significantly elongated in declination; the inertial coordinate representation immediately yields the uncertainty to be expected in the attitude determination for an inertial platform. Figure 2 can be compared to Fig. 10-8 of Ref. 1. When compared to the figure from Ref. 1, the segment from this Note would be a portion of a parallel of the local sphere centered on the spacecraft with the Sun located at the North Pole. However, our figure gives results directly in inertial coordinates, which is not the case of Fig. 10-8.

Earth-Mask Viewing Constraint for a Space Telescope

The case of an inertial space platform in a circular high inclination orbit is also considered. It can be readily seen from Fig. 3 that in one orbit the apparent Earth sweeps a band of the celestial sphere. If α_N is the right ascension of the ascending node and i the inclination of the orbit, the location of the pole of the orbit will be $\alpha_P = \alpha_N - \pi/2$, $\delta_P = \pi/2 - i$. The boundaries of the Earth-mask path are defined by their elevation angles θ_i, θ_s counted from axis OP. If the space telescope orientation θ_T as measured from OP is such that $\theta_i < \theta_T < \theta_s$, the space telescope will suffer a partial eclipse. As time elapses, the pole of the orbit moves on a parallel, and our representation conditions of eclipse can be obtained easily. Furthermore, additional constraints on star or Sun sensors can be handled with the same representation. For example, the crude attitude determination of the French-Soviet experiment Sigma is obtained with a bright-star-large-field-of-view sensor and a slit Sun sensor. Solution in celestial coordinates can be obtained easily since all the necessary information is expressed in celestial coordinates.

Reference

[1]Chen, L.C. and Wertz, J.R., "Geometrical Basis of Attitude Determination," *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Co., Dordrecht, The Netherlands, 1978, pp. 343-361.

Roll Damping of Cruciform-Tailed Missiles

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Nomenclature

C_ℓ	= rolling moment coefficient, rolling moment/qSD
C_{ℓ_p}	= roll damping coefficient, $\partial C_\ell/\partial(pD/2V)$
C_{ℓ_δ}	= rolling moment due to cant of tail surfaces, $\partial C_\ell/\partial\delta$
D	= reference length
M	= freestream Mach number
p	= roll rate, rad/s
$(pD/2V)$	= reduced frequency
q	= freestream dynamic pressure
S	= reference area
V	= freestream velocity
Y_{CENT}	= radial distance from body centerline to centroid of area of exposed tail fin
δ	= cant angle of one fin (all fins deflected uniformly), rad

Introduction

A simple method for predicting roll damping derivatives of cruciform-tailed missiles is presented here. The method is based on an empirical correlation of experimenal data for several cruciform-tailed missiles,[1-8] at Mach numbers from 0 to 4.0. Its use is limited to small angles of attack.

Prediction Method

Configurations for which experimental data were correlated are shown at the top of Fig. 1. Roll damping is correlated in Fig. 1 with the rolling moment due to tail cant angle and the distance from the missile centerline to the centroid of the exposed fin area. The dashed line represents the approximate average of much data for the Basic Finner missile. The Boeing missile was tested with five different sets of fins. The range and average of results for the five configurations are shown. Based on this correlation, the following equation can be used to predict roll damping.

$$C_{\ell_p} = -2.15(Y_{\text{CENT}}/D)C_{\ell_\delta} \tag{1}$$

For this equation to be valid, the roll rate reduced frequency $(pD/2V)$ must include the number 2, and the same reference length must be used for the rolling moment coefficient and reduced frequency.

The type of correlation shown in Fig. 1 is not new. Bolz and Nicolaides[1] used supersonic linearized theory to predict $(C_{\ell_p}/C_{\ell_\delta})$ for the Basic Finner missile and compared predictions with experimental data. Adams and Dugan,[9] using slender body theory, predicted that $(C_{\ell_p}/C_{\ell_\delta})$ was a function of the ratio of body diameter to tail span. However, the experimental data in Fig. 1 would not correlate using the correlation curve developed by them.

Introduction of the empirical term (Y_{CENT}/D) provides an excellent correlation of roll damping data for a wide range of

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